Journal of Libyan Academy Bani Walid **2026**
مجلة الأكاديمية الليبية بني وليد
e-ISSN: 3104-3860
Volume 2, Issue 2, 2026, Pages: 227-246
Website: https://journals.labjournal.ly/index.php/Jlabw/index

Assess the performance of the ARIMA and ARIMAX models to forecast time series data.

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تقييم أداء نموذج *ARIMA* ونموذج *ARIMAX* للتنبؤ ببيانات السلاسل الزمنية.

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Received: 20-01-2026 | **Accepted:** 20-03-2026 | **Published**: 02-04-2026

Abstract

This study evaluates the forecasting performance of time series models for predicting the price of 18-carat broken gold in the parallel market of Tripoli, Libya. While the ARIMA model is widely used for univariate time series forecasting, it often fails to account for external factors. In contrast, the ARIMAX model incorporates exogenous variables, potentially improving accuracy. This research compares four models: ARIMA, ARIMAX, a simple linear regression model, and a mixed model (combining ARIMA with residuals from an external variable model). The dataset spans from February 8, 2019, to April 30, 2019, with the dollar price as the exogenous variable. The results indicate that the ARIMAX(1,2,0)(1,1,0) model outperforms the others, demonstrating superior forecasting accuracy. The mixed model (ARIMA + residuals) ranks second, followed by the ARIMA(1,2,0) model, while the simple linear regression performs the worst. These findings are validated using RMSE and R^2 metrics. The study concludes that incorporating external variables, as in ARIMAX, significantly enhances gold price forecasting, making it a preferred approach over traditional univariate models.

Keywords: Time series, ARIMA model, ARIMAX model, simple linear regression model .

الملخص

تُقيّم هذه الدراسة أداء نماذج السلاسل الزمنية في التنبؤ بسعر كسر الذهب عيار 18 قيراطًا في السوق الموازية بطرابلس، ليبيا. في حين يُستخدم نموذج **ARIMA** على نطاق واسع للتنبؤ بالسلاسل الزمنية أحادية المتغير، إلا أنه غالبًا ما يفشل في مراعاة العوامل الخارجية. في المقابل، يتضمن نموذج **ARIMAX** متغيرات خارجية، مما قد يُحسّن الدقة. يُقارن هذا البحث أربعة نماذج: **ARIMA**، **ARIMAX**، ونموذج الانحدار الخطي البسيط، والنموذج المختلط (الذي يجمع بين **ARIMA** وبقايا نموذج متغير خارجي). تمتد مجموعة البيانات من 8 فبراير 2019 إلى 30 أبريل 2019، مع سعر الدولار كمتغير خارجي.

تشير النتائج إلى أن نموذج **ARIMAX(1,2,0)(1,1,0)** يتفوق على النماذج الأخرى، مما يُظهر دقة تنبؤ فائقة. يحتل النموذج المختلط (**ARIMA** + البواقي) المرتبة الثانية، يليه نموذج **ARIMA(1,2,0)**، بينما يُحقق الانحدار الخطي البسيط أسوأ أداء. وقد تم التحقق من صحة هذه النتائج باستخدام مقاييس **RMSE** و **R²** وخلصت الدراسة إلى أن دمج المتغيرات الخارجية، كما هو الحال في نموذج **ARIMAX**، يُحسّن بشكل كبير من التنبؤ بأسعار الذهب، مما يجعله النهج المُفضّل مقارنةً بالنماذج التقليدية أحادية المتغير.

الكلمات المفتاحية: السلاسل الزمنية، نموذج ARIMA، نموذج ARIMAX، نموذج الانحدار الخطي البسيط.

Introduction:

Time series analysis is a fundamental statistical topic widely applied in various fields through mathematical and statistical procedures. Many of these analyses provide crucial estimation functions and contribute significantly to decision-making processes. Additionally, they help in fitting mathematical and statistical models to the studied problem. There are multiple methods and techniques for analyzing and forecasting time series. One of the most commonly used approaches is the autoregressive integrated moving average (***ARIMA***) model (Box *et al.*, 2008). This methodology utilizes historical data from a univariate time series to analyze trends and predict future cycles (Peter & Silvia, 2012). However, time series are often influenced by external factors, and since ARIMA models rely on a single time series without incorporating information from related series, there is a need for a forecasting model that integrates multiple time series and captures the dynamic characteristics of the system (Chaleampong & Kruangpradit, 2013). This model, known as the autoregressive integrated moving average with explanatory variables (***ARIMAX***) model (Box & Tiao, 1975), belongs to the class of transfer function models. It presents a promising alternative to traditional ***ARIMA*** models by combining historical data with regression-based explanatory variables while accounting for errors inherent in forecasting models.

In this study, we will employ a simple linear regression model, an ***ARIMA*** model, an ***ARIMAX*** model, and a mixed model to analyze and forecast the price series of 18-carat gold in the parallel market. Our objective is to evaluate whether the ***ARIMAX*** model delivers the most accurate results.

Time Series Models:

Autoregressive Integrated Moving Average Models (*ARIMA* Models):

The autoregressive integrated moving average ***(ARIMA)***) model can be regarded as an extension of the ***ARMA*** model. The process y_t is said to be an autoregressive integrated moving average process, ***ARIMA(p,d,q)***, if $z_t = \nabla^d y_t = (1-B)^d\, y_t$ is a stationary ***ARMA (p,q)*** process. In general, we can write the model as

$$\phi_p(B)\nabla^d y_t = \theta_q(B)\varepsilon_t\,,\ \ \varepsilon_t \sim WN(0,\sigma^2)$$

where:

$\boldsymbol{\phi_p(B)}$ is a stationary autoregressive operator. $\boldsymbol{\theta_q(B)}$ is a stationary moving average operator. ***d*** is the number of differencing done to the series to achieve stationarity. $\boldsymbol{\nabla = 1\text{-}B}$ is the difference operator.

Box-Jenkins has specified four stages for ***ARIMA*** model selection (Ababio, 2012; AlAjez,2016 ; Enaami *et at*., 2024); Model Identification (Selecting an Initial Model), Model Estimation, Model Checking and Forecasting with the Model.

Autoregressive Integrated Moving Average with Explanatory Variable Models (*ARIMAX Models)*:

The ***ARIMAX*** model is a generalization of the ***ARIMA*** model, which is capable of incorporating an external input variable. Assume two time series denoted $\boldsymbol{y_t}$ and $\boldsymbol{x_t}$, which are both stationary (Wei, 1990). Then, the ***ARIMAX*** model can be written as follows:

$$\boldsymbol{y_t = \frac{w(B)B^b}{\delta(B)} x_t + \frac{\theta(B)}{\phi(B)} \varepsilon_t \qquad , \varepsilon_t \sim WN(0, \sigma^2_{\varepsilon}.\ )}$$
$$\boldsymbol{= v(B)x_t + n_t}$$

where:

$\boldsymbol{x_t}$ is the input series (independent variable). $\boldsymbol{y_t}$ is the output series (dependent variable).

$\boldsymbol{n_t = \frac{\theta(B)}{\phi(B)} \varepsilon_t}$ is the noise series of the system that is independent of the input series $\boldsymbol{x_t}$.

$\boldsymbol{v(B) = \frac{w(B)B^b}{\delta(B)}}$ is the transfer function.

$\boldsymbol{w(B)}$ and $\boldsymbol{\delta(B)}$ are the transfer function weights.

$$\boldsymbol{w(B) = w_0 - w_1 B - \ \dots\dots. -w_S B^S}$$
$$\boldsymbol{\delta(B) = 1 - \delta_1 B - \ \dots\dots\dots. -\delta_r B^r}$$

(s, r, b) are the parameters of the transfer function. ***b*** means the delay time or number of time units before $\boldsymbol{x_t}$ begins to affect $\boldsymbol{y_t}$ is ***b*** time units.

r means that $\boldsymbol{y_t}$ is affected by its previous value until ***r*** slows down, i.e., $\boldsymbol{y_t}$ is affected by $\boldsymbol{y_{t-1}}$, $\boldsymbol{y_{t-2}, \dots, y_{t-r}}$.

s means that the new value of $\boldsymbol{x_t}$ will continue to affect $\boldsymbol{y_t}$ for ***s*** number of time periods, or in other words, $\boldsymbol{y_t}$ is affected by values from $\boldsymbol{x_{t-b-s}, \dots, x_{t-b}}$.

B is a backshift operator $\boldsymbol{Bx_t = x_{t-1}}$.

The Cross-Correlation Function (*CCF*):

The cross-correlation function is a useful measure of strength and direction of correlation between two random variables (Wei, 1990). For a given set of time series data $\boldsymbol{x_t}$ and $\boldsymbol{y_t}$, $\boldsymbol{1 \le t \le n}$, the cross-correlation function $\boldsymbol{\rho_{xy}(k) = \frac{\gamma_{xy}(k)}{\sigma_x\, \sigma_y}}$, $\boldsymbol{k = 0, \pm 1, \pm 2, \dots}$ is estimated by the sample cross-correlation function (***SCCF***)

$$\boldsymbol{\hat{\rho}_{xy}(k) = \frac{\hat{\gamma}_{xy}(k)}{S_x\, S_y} \qquad , k = 0, \pm 1, \pm 2, \dots}$$

where:

$\boldsymbol{\gamma_{xy}(k) = E[(x_t - \mu_x)][(y_{t+k} - \mu_y)]}$, $\boldsymbol{k = 0, \pm 1, \pm 2, \dots\dots}$ is the cross-covariance function between $\boldsymbol{x_t}$ and $\boldsymbol{y_t}$.

$\boldsymbol{\sigma_x}$ and $\boldsymbol{\sigma_y}$ are the standard deviations of the $\boldsymbol{x_t}$ and $\boldsymbol{y_t}$ series, respectively.

$$\hat{\gamma}_{xy}(k) = \begin{cases} \frac{1}{n}\sum_{t=1}^{n-k}(x_t-\bar{x})(y_{t+k}-\bar{y}), & k \ge 0 \\ \frac{1}{n}\sum_{t=1-k}^{n}(x_t-\bar{x})(y_{t+k}-\bar{y}), & k < 0 \end{cases}$$

$$S_x = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(x_t-\bar{x})^2} \qquad , S_y = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t-\bar{y})^2}$$

and $\bar{x}$ and $\bar{y}$ are the sample means of the x_t and y_t series, respectively.

Construction of the *ARIMAX* Model:

The ***ARIMAX*** model is constructed in four steps as follows (Josué, *et al*., 2023 ; Amiri, *et al*., 2025):

Step 1: **Model Identification:**

The transfer function ***v(B)*** is obtained from the following simple steps:

1. Prewhiten the input series. $\emptyset_x(B)x_t = \theta_x(B)\,\alpha_t$

, where α_t is a white noise series with mean ***zero*** and variance σ_α^2.$\alpha_t = \frac{\emptyset_x(B)}{\theta_x(B)}\, x_t$

2. Calculate the filtered output series. That is, transform the output series y_t using the above prewhitening model to generate the series.

$$\beta_t = \frac{\emptyset_x(B)}{\theta_x(B)}\, y_t$$

3. Determine the order of the transfer function (***s,r,b***) as follows:
   - Through the sample cross-correlation function (***SCCF***) graph between α_t and β_t.
   - If it is not possible to determine the values of ***(s,r,b)*** from the ***SCCF*** between $\hat{\alpha}_t$ and $\hat{\beta}_t$, several sets of values can be tried, and the set with the lowest statistical criteria can be chosen.

Once ***s, r***, and ***b*** are chosen, we can find initial estimates of $\hat{w}(B)$ and $\hat{\delta}(B)$. Thus, we have a first estimate of the transfer function ***v(B)*** as

$$\hat{v}(B) = \frac{\hat{w}(B)}{\hat{\delta}(B)}\, B^b$$

Identification of the Noise Model Once we obtain the preliminary transfer function, we can calculate the estimated noise series.

$$\hat{n}_t = y_t - \hat{v}(B)x_t = y_t - \frac{\hat{w}(B)}{\hat{\delta}(B)}\, B^b\, x_t$$

The appropriate model for the noise can then be identified by examining its sample ***ACF*** and ***PACF***. $\emptyset(B)\, n_t = \theta(B)\, \varepsilon_t$

Step 2: Model Estimation:

After identifying a tentative ***ARIMAX*** model

$$y_t = -\frac{w(B)}{\delta(B)}\, x_{t-b} + \frac{\theta(B)}{\emptyset(B)}\, \varepsilon_t$$

We need to estimate the parameters $\delta = (\delta_1, \delta_2, \ldots\ldots, \delta_r)'$, $w = (w_0, w_1, \ldots\ldots, w_s)'$, $\emptyset = (\emptyset_1, \emptyset_2, \ldots\ldots, \emptyset_p)'$, $\theta = (\theta_1, \theta_2, \ldots\ldots, \theta_q)'$ and σ_ε^2

We can rewrite the previous model as $\delta(B)\emptyset(B)y_t = \emptyset(B)w(B)x_{t-b} + \delta(B)\theta(B)\varepsilon_t$

or, equivalently, $c(B)y_t = d(B)x_{t-b} + e(B)\varepsilon_t$

Step 3: Model Checking:

After the model has been identified and its parameters estimated, it is necessary to check the model adequacy before we can use it for forecasting. In the transfer function model, we assume that the $\boldsymbol{\varepsilon_t}$ are white noise and are independent of the input series $\boldsymbol{x_t}$ and hence are also independent of the prewhitened input series $\boldsymbol{\alpha_t}$**.** Thus, in the diagnostic checking of a transfer function model, we have to examine the residuals $\hat{\boldsymbol{\varepsilon}}_t$ from the noise model as well as the residuals $\boldsymbol{\alpha_t}$ from the prewhitened input model to see whether the assumptions hold.

1. Cross-correlation check: To check whether the noise series $\boldsymbol{\varepsilon_t}$ and the input series $\boldsymbol{x_t}$ are independent. For an adequate model, the sample $\boldsymbol{CCF}$, $\hat{\boldsymbol{\rho}}_{\alpha\hat{\varepsilon}}(\mathbf{k})$**,** between $\hat{\boldsymbol{\varepsilon}}_t$ and $\boldsymbol{\alpha_t}$ should show no patterns and lie within their two standard errors $\mathbf{2(n-k)^{-1/2}}$.
2. Autocorrelation check: To check whether the noise model is adequate. For an adequate model, both the sample $\boldsymbol{ACF}$ and $\boldsymbol{PACF}$ of $\hat{\boldsymbol{\varepsilon}}_t$ should not show any patterns.

Step 4: Forecasting with the Model:

Once a transfer function model is found to be adequate, it can be used to improve the forecast of the output series $\boldsymbol{y_t}$, by using the past history of both the output series $\boldsymbol{y_t}$, and the associated Input series $\boldsymbol{x_t}$ (Sharma, 2016).

Analysis and Results:

This study utilizes 82 daily observations of two variables: the dollar price (*independent variable*) and the price of breaking 18-carat gold (*dependent variable*) in the parallel market of Tripoli, Libya, spanning the period from February 8, 2019, to April 30, 2019.

A statistical summary of both series reveals the following:

- The dollar price series ranged between 4.160 and 4.570, with an arithmetic mean of 4.358 and a standard deviation of 0.107.
- The gold break price series fluctuated between 121.500 and 142.000, with an arithmetic mean of 132.561 and a standard deviation of 3.949.

The higher standard deviation of the gold break price series suggests that it exhibits less homogeneity compared to the dollar price series.

Simple Linear Regression model:

First, we fit our data by using the simple linear regression model Montgomery *et al.,* 2012). Looking at Figure (1.1), which shows the scatter plot of this data, there appears to be a moderately positive linear relationship between the independent variable (dollar prices) and the dependent variable (gold break prices). It is clear that the determination coefficient $\boldsymbol{R^2}$= 0.429, meaning that 0.429 explains the percentage of changes that occur in the dependent variable $\boldsymbol{y_t}$ resulting from changes that occur in the independent variable $\boldsymbol{x_t}$.

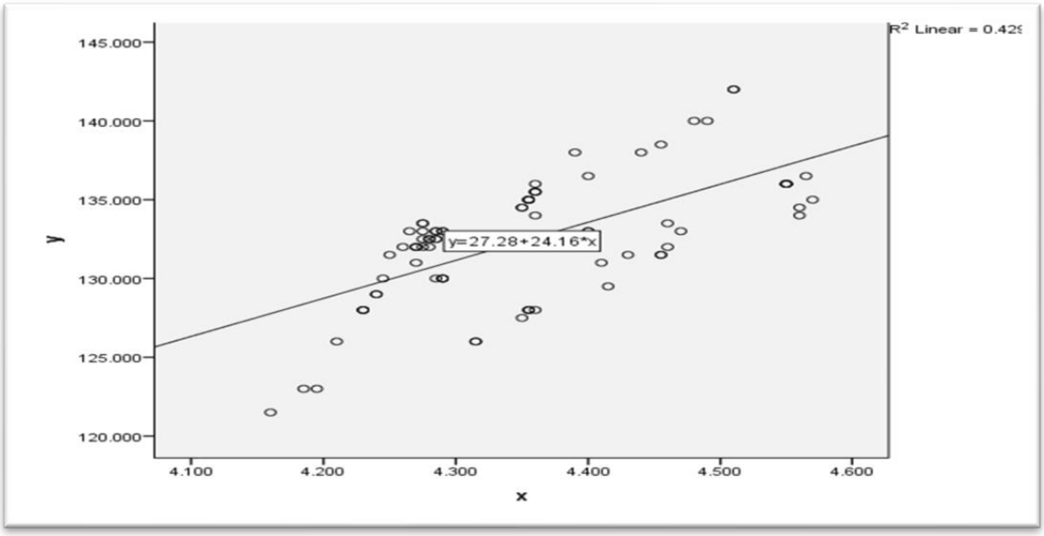
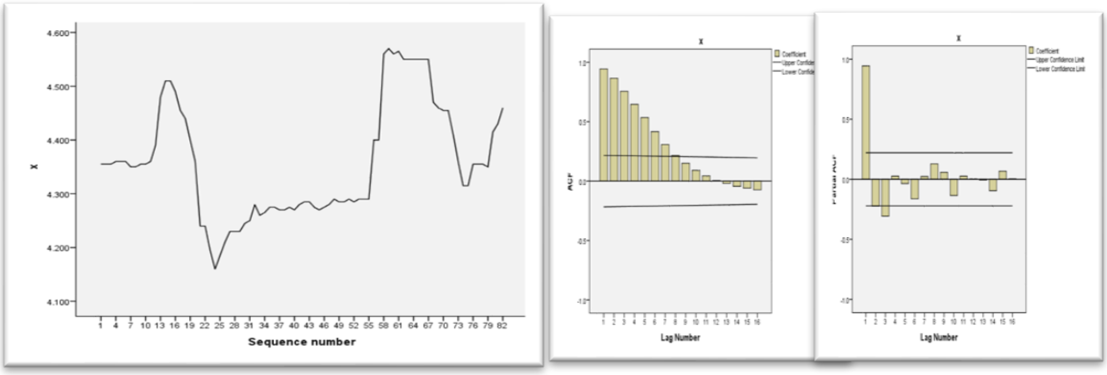


Figure (1): scatter plot

The **regression coefficient** for the independent variable (**dollar price**) is 24.158, indicating that, under the assumption that all other variables stay constant, the gold break price should rise by 24.158 units for every one unit increase in the dollar price. The ***p-value*** of the regression coefficient is 0.000, which indicates that the relationship between dollar price and gold break price is **statistically significant**. The equation of the regression line is as follows: $\hat{y}_t = 27.276+24.158\ x_t$

ARIMA MODEL:

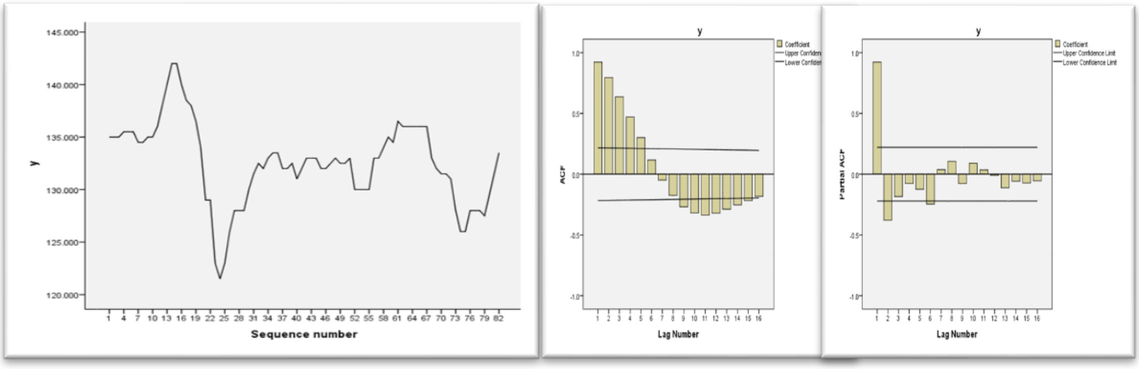
Secondly, we fit our data by using the ***ARIMA*** model. By looking at Figure (1.2), plot (a), which shows the dollar price series, it is evident that the levels of price don't appear to be stationary. In contrast, Plot (b), the autocorrelation function (***ACF***), shows the data is dying slowly, which means that we are dealing with a non-stationary time series, and Plot (c), the partial autocorrelation function (***PACF***), shows the data has a cutoff after the third lag.



(a) **(b)** **(c)**

Figure (2): (a) The series of daily prices of Dollar x_t, (b) *ACF*, and (c) *PACF*

By looking at Figure (1.3), plot (a), which shows the gold break price series, it is evident that the levels of price don't appear to be stationary. In contrast, Plot (b), the autocorrelation function (***ACF***), shows the data as a sine wave function; furthermore, the partial autocorrelation function (***PACF***) in Plot (c) shows that there is more than one cut-off at the second and sixth lags. This means that we are dealing with a non-stationary time series.



(a) **(b)** **(c)**

Figure (3): (a) The series of daily prices of gold break y_t, (b) *ACF*, and (c) *PACF*

To further confirm the non-stationary nature of the two series, as we explained in Figures (1.2) and (1.3), we will use one of the important statistical tools. This tool is called unit root tests.

The Unit Root Tests for Stationary:

To examine the stationarity of the time series, we apply the most common **unit root tests**, which include:

- **Augmented Dickey-Fuller (*ADF*) Test** and **Phillips-Perron (*PP*) Test**: The null hypothesis states that the data is **non-stationary** (Dickey & Fuller, 1981; Phillips & Perron, 1988).
- **Kwiatkowski-Phillips-Schmidt-Shin (*KPSS*) Test**: The null hypothesis states that the data is **stationary** (Kwiatkowski *et al*., 1992).

According to the results for the **dollar price series and** the gold break price series are show in Table (1.1) both series are not stationary.

Table (1): Results of Unit Root Tests for dollar price series and Gold break price series

Test for dollar price	**P-value**	**Decision**	**Test for Gold break price**	**P-value**	**Decision**
PP	**0.580**	Accepted H_0 (non-stationary)	**PP**	**0.410**	Accepted H_0 (non-stationary)
ADF	**0.370**	Accepted H_0 (non-stationary)	**ADF**	**0.080**	Accepted H_0 (non-stationary)
KPSS	**0.020**	Accepted H_1 (non-stationary)	**KPSS**	**0.090**	Accepted H_0 (non-stationary)

Differencing for Stationarity:

To ensure stationarity in the time series, we apply the Box-Jenkins technique, which involves transforming the data through first differencing or second differencing**.**

The initial approach involves taking the first difference and performing unit root tests to assess stationarity. The results indicate the following:

- ***PP*** test *p-value*: **0.010** (less than 0.05)
- ***KPSS*** test *p-value*: **0.100** (greater than 0.05)

These results suggest that both the dollar price series and gold break price series are stationary. To further address stationarity, a second difference **(d = 2)** is taken. We confirm that the dollar price and gold break price series have successfully achieved stationarity.

Building *ARIMA* models for stationary series:

(I) Models Identification:

Identification of an ***AR*** model is often best done with the ***PACF***, and identification of an ***MA*** model is often best done with the ***ACF***. Our aim now is to find an appropriate ***ARIMA*** model based on the ***ACF*** and ***PACF***. It seems possible that ***AR(1)*** is the best fit based on the one spike in the first lag of the ***PACF*** plot for the two stationary series $x_t^{``}$ and $y_t^{``}$. On the other hand, it seems possible that ***MA(1)*** is the best fit based on the one spike in the first lag of the ***ACF*** plot for the two stationary series $x_t^{``}$ and $y_t^{``}$. This primary analysis proposes that the models suggested for the two stationary series $x_t^{``}$ and $y_t^{``}$ are ***ARIMA(1,2,0)***, ***ARIMA(0,2,1)***, and ***ARIMA(1,2,1)***.

The strategy used in selecting the appropriate model from suggested models is based on the root mean square error ***(RMSE)***, mean absolute percent error ***(MAPE)***, Bayesian information criteria ***(BIC)***, and mean absolute error ***(MAE)*** **(** Ospina, *et al*. (2023)**).** Table (1.2) gives the suggested models with their respective fit statistics.

Table (2): The values of *RMSE*, *MAPE*, *BIC*, and *MAE* for suggested *ARIMA* models

MODEL	Series $x_t^{``}$				series $y_t^{``}$			
	RMSE	***MAPE***	***BIC***	***MAE***	***RMSE***	***MAPE***	***BIC***	***MAE***
ARIMA(1,2,0)	0.035	0.486	-6.661	0.021	1.546	0.858	0.926	1.122
ARIMA(0,2,1)	0.035	0.518	-6.629	0.023	1.613	0.919	1.011	1.202
ARIMA(1,2,1)	0.035	0.489	-6.598	0.021	1.491	0.800	0.908	1.048

The results in Table (1.2) shows that for the series $x_t^{``}$, it is clear that the ***ARIMA(1,2,0) model*** has the lowest ***MAPE*** (0.486) and ***BIC*** (-6.661), while it shares the lowest value of ***RMSE*** (0.035) with the two models ***ARIMA(0,2,1)*** and ***ARIMA(1,2,1)*** and shares the lowest value of ***MAE*** (0.021) with the model ***ARIMA(1,2,1)***. Therefore, the suggested model is ***ARIMA(1,2,0)***. As for the series $y_t^{``}$, it is clear that the model ***ARIMA(1,2,1)*** has the lowest values of ***RMSE*** (1.491), ***MAPE*** (0.800), ***BIC*** (0.908), and ***MAE*** (1.048). Therefore, it is the suggested model. Now, we will estimate the parameters of the suggested models.

(II) Models Estimation:

To estimate the parameters of the suggested models ***ARIMA(1,2,0)*** and ***ARIMA(1,2,1),*** we have the following results:

Table (3): Parameter estimation for the *ARIMA(1,2,0)* model for the series $x_t^{``}$

MODEL			series $x_t^{``}$		
			Estimate		***P-value***
ARIMA(1,2,0)	***AR***	Lag1	ϕ_1	-0.549	0.000

From Table (3), the ***p-value*** of $\boldsymbol{\phi_1}$ is less than 0.05, so the parameter is significant, and we conclude that the model is appropriate.

Table (4): Estimating parameters for the *ARIMA(1,2,1)* model for the series $y_t^{``}$

MODEL			series $y_t^{``}$		
			Estimate		***P-value***
ARIMA(1,2,1)	***AR***	Lag1	ϕ_1	0.426	0.001
	MA	Lag1	θ_1	1.000	0.893

From Table (4), the ***p-value*** of $\boldsymbol{\phi_1}$ is less than 0.05, so the parameter is significant, but the ***p-value*** of the parameter $\boldsymbol{\theta_1}$ is greater than 0.05, so the parameter is not significant, and we conclude that the model is not appropriate.

Therefore, we will estimate the parameters of the model that follows immediately in terms of the lowest statistical criteria, which is the ***ARIMA(1,2,0) model***, as follows:

Table (5): Parameter estimation for the *ARIMA(1,2,0)* model for the series $y_t^{``}$

MODEL			series $y_t^{``}$		
			Estimate		***P-value***
ARIMA(1,2,0)	***AR***	Lag1	ϕ_1	-0.392	0.000

From Table (5), the ***p-value*** of $\boldsymbol{\phi_1}$ is less than 0.05, so the parameter is significant, and we conclude that the model is appropriate. At this point, it can be said that, amongst all the suggested models, ***ARIMA(1,2,0)*** for $\boldsymbol{x_t^{``}}$ and ***ARIMA(1,2,0)*** for $\boldsymbol{y_t^{``}}$ are the best fit models. Fitted models in this case are:

$$(1+0.549B)(1-B)^2\hat{x}_t=\alpha_t \qquad ,\alpha_t \sim WN(0,0.035)$$
$$(1+0.392B)(1-B)^2\hat{y}_t=\beta_t \qquad ,\beta_t \sim WN(0,1.613)$$

(III) Model Checking:

In time series modeling, choosing the best model to fit the data is directly related to whether the residual analysis was performed well. We can check the properties of the residual by:

1- Check the mean of the residuals by considering the **one-sample t-test** of the residuals of the two models, ***ARIMA(1,2,0)*** for $\boldsymbol{x_t^{``}}$ and ***ARIMA(1,2,0)*** for $\boldsymbol{y_t^{``}}$.
2- Check the randomness of the residuals by considering the **Runs test** about zero of the residuals of the two models, ***ARIMA(1,2,0)*** for $\boldsymbol{x_t^{``}}$ and ***ARIMA(1,2,0)*** for $\boldsymbol{y_t^{``}}$.

3- Check the autocorrelation problem of the residuals by considering the graph of ***ACF*** and ***PACF*** of the residuals of the two models, ***ARIMA(1,2,0)*** for $\boldsymbol{x_t}^{``}$ and ***ARIMA(1,2,0)*** for $\boldsymbol{y_t}^{``}$.

4- Check the normality of the residuals by considering the graph of the ***P-P*** normal for the residuals of the two models, ***ARIMA(1,2,0)*** for $\boldsymbol{x_t}^{``}$ and ***ARIMA(1,2,0)*** for $\boldsymbol{y_t}^{``}$.

Firstly, from Table (6), the ***p-values*** of the **one-sample t-test** for the residuals of ***ARIMA(1,2,0)*** for $\boldsymbol{x_t}^{``}$ and ***ARIMA(1,2,0)*** for $\boldsymbol{y_t}^{``}$ are equal to 0.904 and 0.848, respectively, which are both greater than 0.05; therefore, we cannot reject the null hypothesis that the mean of these residuals is zero.

Table (6): one-sample t-test for the residuals of the two models, *ARIMA(1,2,0)* for $x_t^{``}$ and *ARIMA(1,2,0)* for $y_t^{``}$

One-Sample t-test	**Test Value = 0**		
	t	***Df***	***p-value (2-tailed)***
Residuals of the *ARIMA(1,2,0)* for $\boldsymbol{x_t}^{``}$	0.121	79	0.904
Residuals of the *ARIMA(1,2,0)* for $\boldsymbol{y_t}^{``}$	0.193	79	0.848

Secondly, from Table (7), the ***p-values*** of the **Runs test** for the residuals of ***ARIMA(1,2,0)*** for $\boldsymbol{x_t}^{``}$ and ***ARIMA(1,2,0)*** for $\boldsymbol{y_t}^{``}$ are equal to 0.585 and 0.959, respectively, which are both greater than 0.05; therefore, we cannot reject the null hypothesis that these residuals are random.

Table (7): Runs test for the residuals of the two models, *ARIMA(1,2,0)* for $x_t^{``}$ and *ARIMA(1,2,0)* for $y_t^{``}$

Runs test	***p-value***
Residuals of the *ARIMA(1,2,0)* for $\boldsymbol{x_t}^{``}$	0.585
Residuals of the *ARIMA(1,2,0)* for $\boldsymbol{y_t}^{``}$	0.959

Thirdly, in Figure (4), we show that (a) the ***ACF*** and ***PACF*** to the ***ARIMA(1,2,0)*** for $\boldsymbol{x_t}^{``}$ and (b) the ***ACF*** and ***PACF*** to the ***ARIMA(1,2,0)*** for $\boldsymbol{y_t}^{``}$ do not have any significant lags, which indicates that the models are good models to represent our data.

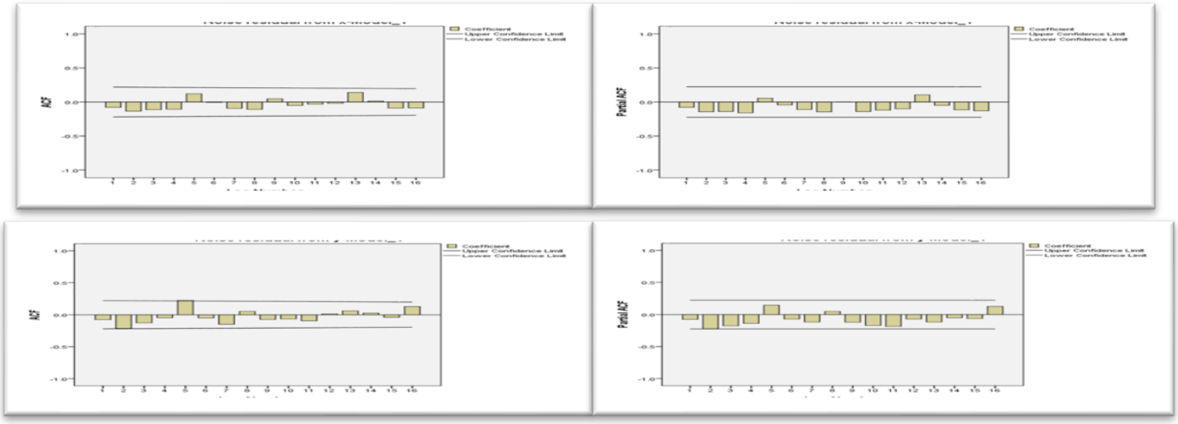
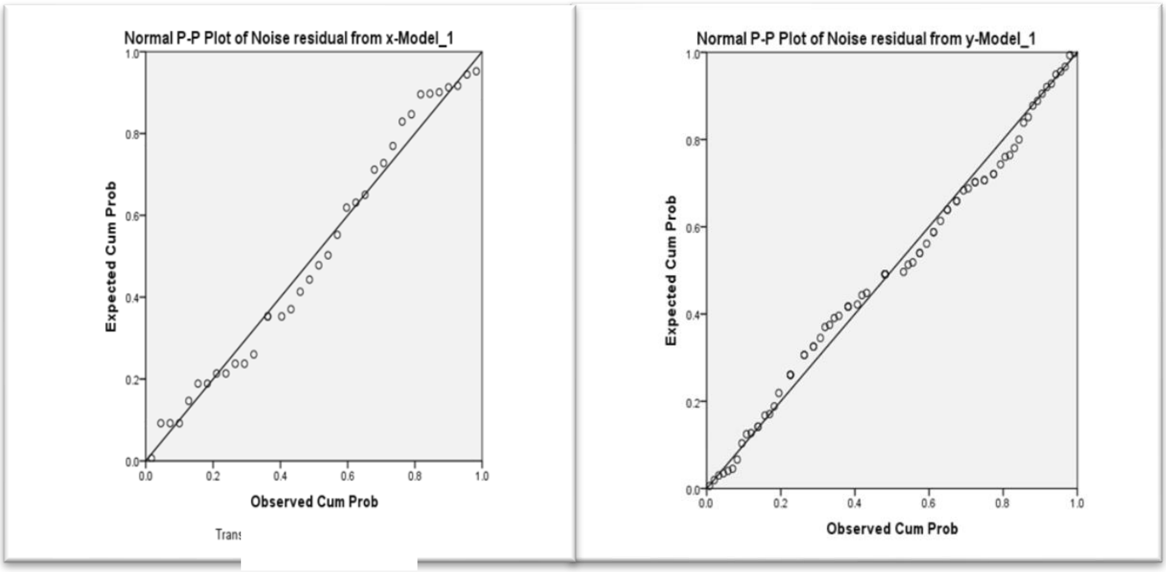


Figure (4): (a) the ***ACF*** and ***PACF*** to the ***ARIMA(1,2,0)*** for $\boldsymbol{x_t}^{``}$, and (b) the ***ACF*** and ***PACF*** to the ***ARIMA(1,2,0)*** for $\boldsymbol{y_t}^{``}$

Finally, for more diagnostic checking, we can look at Figure (5), which shows that (a) the ***P-P* plot** of the normality test for the residuals of ***ARIMA(1,2,0)*** for $x_t^{``}$ and (b) the ***P-P* plot** of the normality test for the residuals of ***ARIMA(1,2,0)*** for $y_t^{``}$. We note that the points closely follow the straight line, indicating that the residuals follow a normal distribution.



(a) **(b)**

Figure (5): The *P-P* plot for the residuals of ***ARIMA(1,2,0)*** for $x_t^{``}$ and ***ARIMA(1,2,0)*** for $y_t^{``}$.

The above concludes that the ***ARIMA(1, 2, 0)*** model is effective for forecasting the gold break price series, as it successfully passed all examinations and diagnostic tests. However, it often fails to capture the nuances introduced by external factors. In contrast, the ***ARIMAX*** model can capture relationships and dynamics that ***ARIMA*** models may overlook by incorporating external variables into the time series model, enhancing the accuracy of the phenomenon under study.

Construction of *ARIMAX* Model for stationary series $y_t^{``}$:

Below, we will build an ***ARIMAX*** model for the gold break price data series y_t as a dependent variable using the dollar price data series x_t as an independent variable, noting that the stages of building the model take place on the stationary time series $x_t^{``}$, $y_t^{``}$.

(I) Model Identification:

Step 1: Estimation of the best ***ARIMA*** model for stationary series $x_t^{``}$.

This stage was completed, and the estimation results are shown in the previous Table (4.8), where it was concluded that the best ***ARIMA*** model for the dollar price data series is ***ARIMA(1,2,0)***.

$$(1+0.549B)(1-B)^2\hat{x}_t=\alpha_t \quad ,\alpha_t \sim WN(0,0.035)$$

Step 2: In this step, the following is done:

1) Prewhiten of the stationary series $x_t^{``}$

$$\hat{\alpha}_t=(1+0.549B)(1-B)^2\,x_t$$

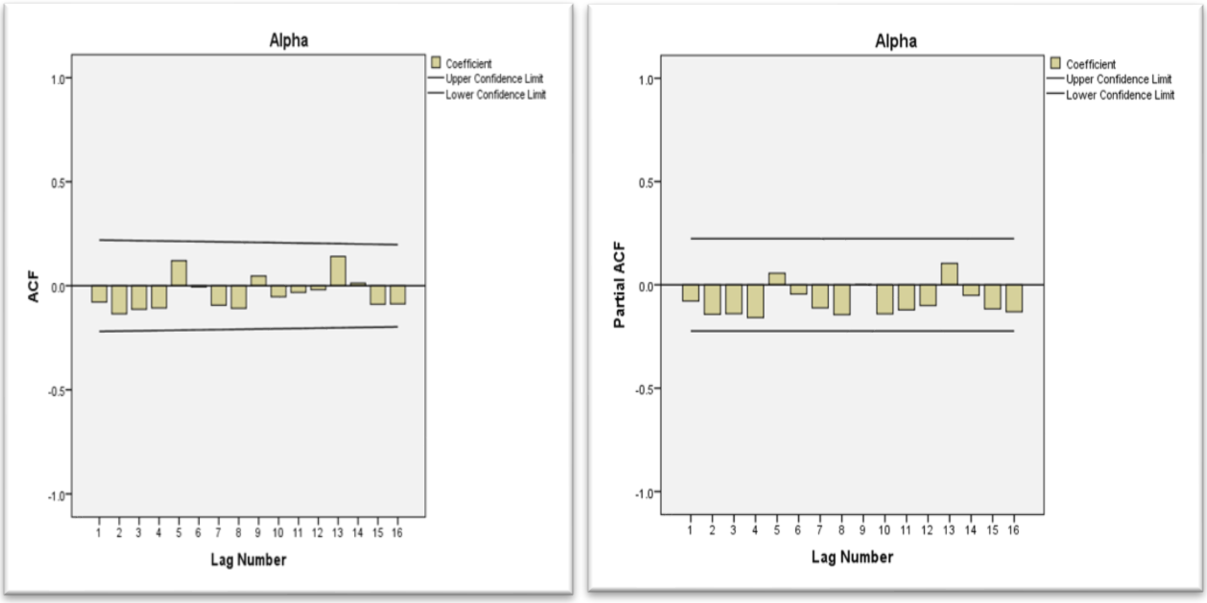


Figure (6): The ACF and $PACF$ for $\boldsymbol{\alpha_t}$

From Figure (6), it is very clear that ACF and $PACF$ do not have any significant lags, which indicates that the stationary series $\boldsymbol{x_t}``$ has been whitened.

2) filter of the stationary series $\boldsymbol{y_t}``$

We will apply the same previous whitening model to the stationary series $\boldsymbol{y_t}``$ to filter it.

$$\hat{\beta}_t = (1 + 0.549B)(1 - B)^2 \, y_t$$

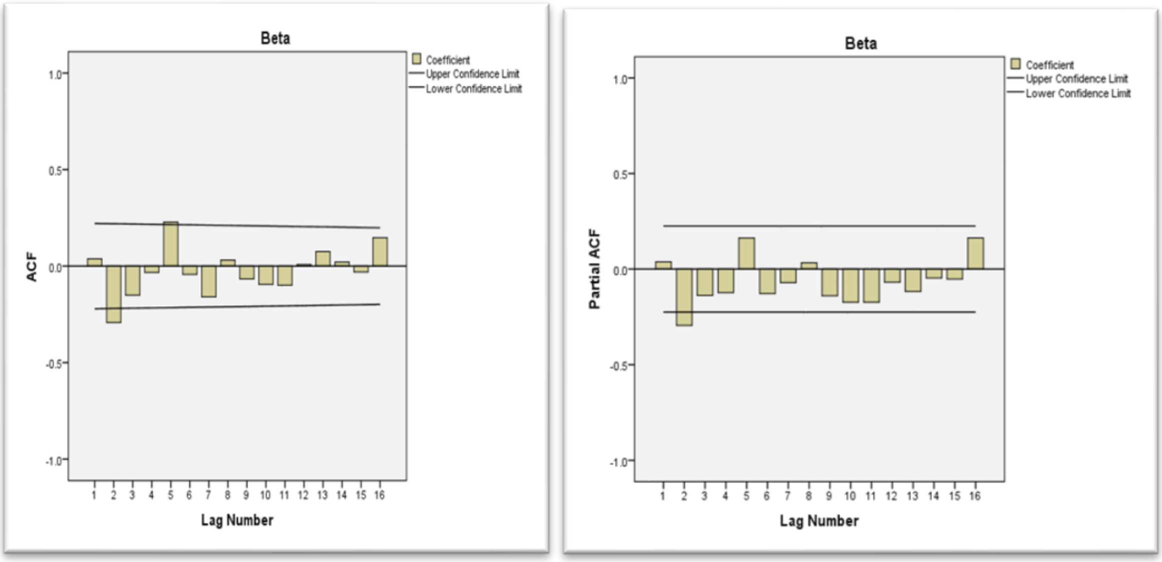


Figure (7): The ACF and $PACF$ for $\boldsymbol{\beta_t}$

From Figure (7), it is obvious that ACF and $PACF$ do have significant lag at lag 2, which indicates that the stationary series $\boldsymbol{y_t}``$ has been filtered; it does not have to be whitened.

3) finding the sample cross-correlation function ***(SCCF)*** between $\hat{\boldsymbol{\alpha}}_t$ and $\hat{\boldsymbol{\beta}}_t$.

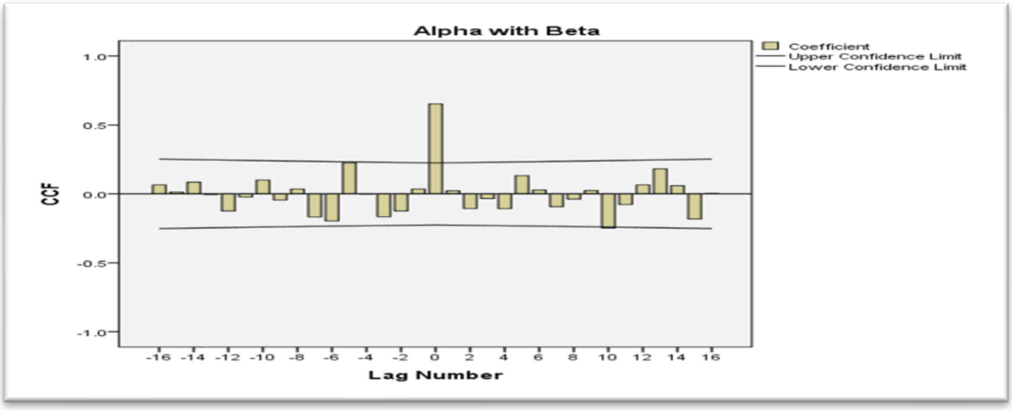


Figure (8): The ***SCCF*** between $\hat{\alpha}_t$ and $\hat{\beta}_t$.

Figure (8) shows the sample cross-correlation function between $\hat{\alpha}_t$, the residuals of the ***ARIMA*** model for the dollar price, and $\hat{\beta}_t$, the residuals of the ***ARIMA*** model for the gold price. From the Figure, we can determine the order of the transfer function (***s, r, b***), as we notice the presence of a significant coefficient at lag 0, and thus, $b = 0$, while we find it difficult to determine ***r*** and ***s***; there is not any pattern to identify them in the sample cross-correlation function.

Step 3:

- Determine the order ***ARIMA*** model of the stationary series $y_t^{``}$, as explained previously. ***ARIMA(p,d,q)*** for $y_t^{``}$ = ***ARIMA(1,2,0)***
- Determine the order of the transfer function ***(s,r,b)*** as follows:
  a) Through the sample cross-correlation function ***(SCCF)*** between $\hat{\alpha}_t$ and $\hat{\beta}_t$ in Figure (1.8), we notice that $b = 0$.
  b) If it is not possible to determine the values of ***s*** and ***r*** from the ***SCCF*** between $\hat{\alpha}_t$ and $\hat{\beta}_t$, several sets of values can be tried, and the set with the lowest statistical criteria can be chosen.

Table (8): The optimal statistical model between the *ARIMAX* models suggested based on the statistical criteria

	MODEL	**Series $y_t^{``}$**			
		RMSE	***MAPE***	***BIC***	***MAE***
(1)	***ARIMAX(1,2,0)(0,1,0)***	1.250	0.682	0.612	0.894
	ARIMAX(1,2,0)(0,2,0)	1.266	0.688	0.696	0.901
	ARIMAX(1,2,0)(0,3,0)	1.280	0.696	0.776	0.911
(2)	***ARIMAX(1,2,0)(1,1,0)***	1.253	0.685	0.672	0.898
	ARIMAX(1,2,0)(1,2,0)	1.269	0.693	0.756	0.907
	ARIMAX(1,2,0)(1,3,0)	1.285	0.697	0.839	0.913
	ARIMAX(1,2,0)(2,1,0)	1.269	0.692	0.756	0.907
	ARIMAX(1,2,0)(2,2,0)	1.276	0.693	0.823	0.907

ARIMAX(1,2,0)(2,3,0)	1.293	0.698	0.909	0.913
ARIMAX(1,2,0)(3,1,0)	1.285	0.697	0.840	0.912
ARIMAX(1,2,0)(3,2,0)	1.293	0.698	0.909	0.914
ARIMAX(1,2,0)(3,3,0)	1.302	0.698	0.980	0.913

The results of Table (1.8) above show that the model ***ARIMAX(1, 2, 0)(0,1,0)*** has the lowest values of ***RMSE*** (1.250), ***MAPE*** (0.682), ***BIC*** (0.612), and ***MAE*** (0.894), followed by the model ***ARIMAX(1,2,0)(1,1,0).***

We note that the order of the numerator of the transfer function in the first model was equal to ***s* = *0***, and the order of the denominator was ***r* =*1***.

Therefore, we will estimate the parameters of the suggested model ***ARIMAX(1,2,0)(0,1,0)***.

(II) Estimating model:

From Table (1.9), we note that the ***p-value*** of the denominator parameter of the **transfer function** is greater than 0.05, which indicates that the parameter is not significant and therefore the model is not appropriate.

Table (9): Parameters estimation for the *ARIMAX(1,2,0)(0,1,0)* model for the series $y_t^{``}$

MODEL			**Series $y_t^{``}$**		
			Estimate		***P-value***
ARIMAX(1,2,0)(0,1,0)	***AR***	Lag1	ϕ_1	-0.453	0.000
	Numerator	Lag0	ω_0	29.991	0.000
	Denominator	Lag1	δ_1	0.124	0.358

Therefore, we will estimate the parameters of the model that follows immediately in terms of the lowest statistical criteria in Table (1.8), which is the ***ARIMAX(1,2,0)(1,1,0)*** model, as follows:

Table (10): Parameters estimation for the *ARIMAX(1,2,0)(1,1,0)* model for the series $y_t^{``}$**.**

MODEL			**Series $y_t^{``}$**		
			Estimate		***P-value***
ARIMAX(1,2,0)(1,1,0)	***AR***	Lag1	ϕ_1	-0.437	0.000
	Numerator	Lag0	ω_0	29.456	0.000
		Lag1	ω_1	-25.621	0.005
	Denominator	Lag1	δ_1	-0.766	0.026

From Table (1.10), the ***p-values*** of $\phi_1, \omega_0, \omega_1,$ and δ_1 are less than 0.05, so the parameters are significant, and we conclude that the model is appropriate. At this point, it can be said that, amongst all the suggested models, ***ARIMAX(1,2,0)(1,1,0)*** for $y_t^{``}$ is the best fit model.

The fitted model in this case is:

$$(1-B)^2 y_t = \frac{w(B)B^b}{\delta(B)}(1-B)^2 x_t + \frac{\theta(B)}{\phi(B)}\varepsilon_t = \frac{(w_0 - w_1 B)}{(1-\delta_1 B)}(1-B)^2 x_t + \frac{\theta(B)}{(1-\phi_1 B)}\varepsilon_t$$

$$(1-B)^2 \hat{y}_t = \frac{(29.456 + 25.621B)}{(1+0.766B)}(1-B)^2 x_t + \frac{1}{(1+0.437B)}\varepsilon_t$$

(III) Model Checking:

The diagnosis is based on examining the residuals and considering the extent to which they achieve the model hypotheses, which are as:

- The ***p-value*** of the **one-sample t-test** for the residuals of ***ARIMAX(1,2,0)(1,1,0)*** for y_t`` is equal to 0.868, which is greater than 0.05; therefore, we cannot reject the null hypothesis that the mean of these residuals is zero.
- Also, the **Runs test** about the mean for the residuals of ***ARIMAX(1,2,0)(1,1,0)*** for y_t`` is equal to 0.735, which is greater than 0.05; therefore, we cannot reject the null hypothesis that these residuals are random.
- **Thirdly**, looking at Figure (1.10), plots (a), (b) show the autocorrelation function ***(ACF)*** and partial autocorrelation function ***(PACF)***, respectively, for the residuals of the model, ***ARIMAX(1,2,0)(1,1,0)*** for y_t``, and it is clear that there is one cut-off after the second lag in plots (a), (b). This means that there are correlations in the residuals.
-

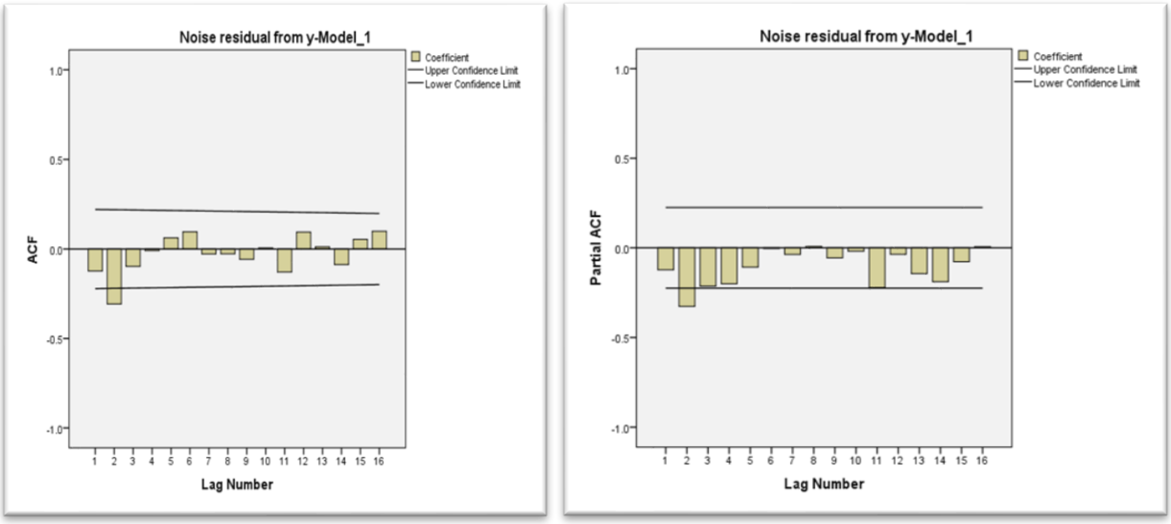


Figure (10): the *ACF* and the *PACF* for the residuals of the model, *ARIMAX(1,2,0)(1,1,0)* for y_t``.

To test whether the correlations of the residuals are significant or not, we do the following:

- **Ljung-Box test** for residuals at lag 2, the ***p-value*** of the **Ljung-Box test** for residuals at lag 2 of the model, ***ARIMAX(1,2,0)(1,1,0),*** for y_t`` is equal to 0.011, which is greater than 0.01; therefore, we cannot reject the null hypothesis that the autocorrelation at the second lag for the residuals of this model is insignificant, which indicates that the model is good to represent our data.

- Estimate the ***ARIMA(2,0,2)*** model for residuals

From Table (11), the ***p-values*** of $\boldsymbol{\phi_1, \phi_2, \theta_1}$, and $\boldsymbol{\theta_2}$ are greater than 0.05, so the parameters are not significant, and we conclude that the model ***ARIMAX(1,2,0)(1,1,0)*** is appropriate (i.e., no more models can be suggested).

- The Coefficient of Determination **($\boldsymbol{R^2}$)** for the ***ARIMAX(1,2,0)(1,1,0)*** model for $\mathbf{y_t}$`` is equal to 0.905; this is a high percentage, which means that the model is suitable for our data.

Table (11): Parameter estimation for the *ARIMA(2,0,2)* model for the residuals of the model, *ARIMAX(1,2,0)(1,1,0)* for y_t``.

MODEL			**residuals series**		
			Estimate		***P-value***
ARIMA(2,0,2)	***AR***	Lag1	ϕ_1	0.213	0.635
		Lag2	ϕ_2	-0.014	0.960
	MA	Lag1	θ_1	0.594	0.565
		Lag2	θ_2	0.402	0.528

Finally, for more diagnostic checking, we can look at the ***P-P*** plot of the normality test for the residuals of ***ARIMAX(1,2,0)(1,1,0)*** for $\mathbf{y_t}$``. We note that the points closely follow the straight line, indicating that the residuals follow a normal distribution.

Building a mixed model for the stationary series $\mathbf{y_t}$`` by adding the residuals of the independent variable model to the dependent variable model as follows:

$$\hat{y}_t = \boldsymbol{ARIMA_{y_t}(1,2,0)} + \boldsymbol{Residual}\,\{\boldsymbol{ARIMA_{x_t}(1,2,0)}\}$$

The fitted model in this case is:

$$\hat{y}_t = 1.608\, y_{t-1} - 0.216\; y_{t-2} - 0.392\, y_{t-3} + \alpha_t$$

Next, we compare the previous four models. The criteria used are the root mean square error ***(RMSE)*** and the coefficient of determination **($\boldsymbol{R^2}$)**.

The results of Table (1.12) show that the model ***ARIMAX(1, 2, 0)(1,1,0)*** has the lowest value of ***RMSE*** (1.253) and the largest value of $\boldsymbol{R^2}$ (0.905), followed by the model $\boldsymbol{ARIMA_{y_t}(1,2,0)}$+ ***Residual*** $\{\boldsymbol{ARIMA_{x_t}(1,2,0)}\}$, then the model ***ARIMA(1,2,0),*** and then the simple linear regression model. Therefore, we can conclude that the ***ARIMAX(1,2,0)(1,1,0)*** model is better than other models.

Table (12): Results of the evaluation for the four types of models based on the order of statistical criteria.

MODEL	series $y_t^{``}$		
	RMSE	$\boldsymbol{R^2}$	**Order**
$\boldsymbol{ARIMAX}$**(1,2,0)(1,1,0)**	1.253	0.905	1
$\boldsymbol{ARIMA_{y_t}(1,2,0)}$**+ Residual** $\boldsymbol{\{ARIMA_{x_t}(1,2,0)\}}$	1.581	0.878	2
$\boldsymbol{ARIMA}$**(1,2,0)**	1.613	0.836	3
Simple Linear Regression	3.001	0.429	4

Forecasting with the *ARIMAX* model for series $\mathbf{y_t}$:

Once the best model is chosen, as shown in Table (4.20), which is the ***ARIMAX(1,2,0)(1,1,0)*** model, and its parameters are estimated, the model is used for forecasting.

We have made in-sample forecasts for the gold break price series after excluding the last three observations from the series, as well as out-of-sample forecasts for a period of 10 days for the same series, displaying the limits of the 95% confidence interval. Figures (1.11), (1.12) show the daily gold break price from February 8, 2019 to April 30, 2019 with 3 days of in-sample forecasts and 10 days of out-of-sample forecasts. it is clear how closely the real values match the estimated values, and this indicates the efficiency of the ***ARIMAX(1,2,0)(1,1,0)*** model to represent our data.

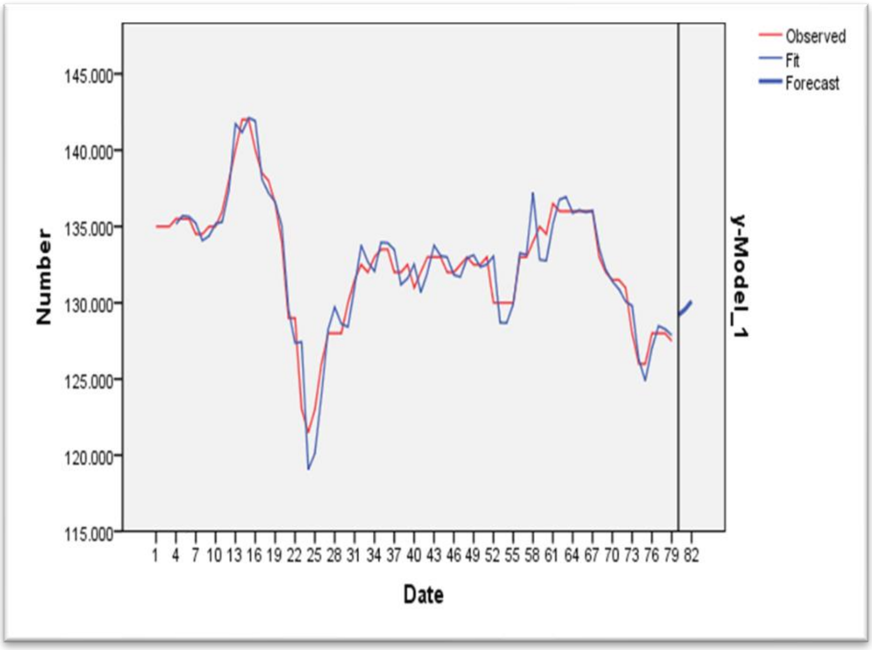


Figure (12): forecasting out-sample

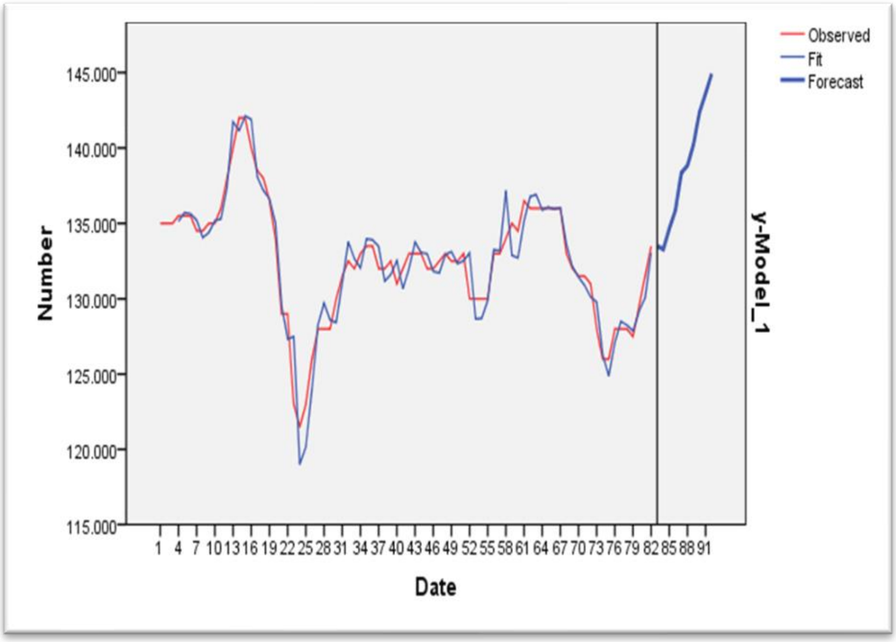


Figure (11): forecasting in-sample

Conclusions:

The primary goal of this study is to shed light on the time series forecasting method using ARIMAX models, which rely on one or more series as inputs (independent variables). As an applied example, we examine forecasting the price of 18-carat gold in the parallel market of Tripoli, Libya, using the dollar price series as an input variable. The gold price series extends from February 8, 2019, to April 30, 2019, representing 82 observations. Similarly, the dollar price series used as an independent variable consists of daily records spanning the same period. During this study, we applied the concepts outlined in the theoretical framework and estimated four statistical models, leading to the following key findings:

- Using a simple linear regression model, the ANOVA Table revealed that the regression is significant. This indicates a somewhat positive linear relationship between the independent variable (dollar prices) and the dependent variable (gold break prices); however, the relationship is not strong, as the coefficient of determination is **$R^2 = 0.429$**.
- Time series plots, autocorrelation functions (ACF and PACF), and unit root tests confirmed that both series are non-stationary. They became stationary after taking the second difference of the original series.
- The best ARIMA model for the gold break price series was determined based on various statistical criteria (RMSE, MAPE, BIC, and MAE). The most suitable model was **ARIMA(1,2,0)**, which successfully passed all diagnostic tests.
- The best ARIMAX model for the gold break price series considering the dollar price series as an independent variable was selected using RMSE, MAPE, BIC, and MAE. The most appropriate model was **ARIMAX(1,2,0)(1,1,0)**, which also passed all diagnostic tests.
- A mixed model for the gold break price series was constructed by incorporating the residuals of the ARIMA model for the dollar price series into the ARIMA model of the gold break price series.

- Comparing the four models using root mean square error (RMSE) and the coefficient of determination (R^2) revealed that **ARIMAX(1,2,0)(1,1,0)** performed best, followed by $\boldsymbol{ARIMA_{y_t}(1,2,0)}$ + Residual $\{\boldsymbol{ARIMA_{x_t}(1,2,0)}\}$, then **ARIMA(1,2,0)**, and finally the simple linear regression model.
- Forecasting was conducted using the ARIMAX model for the gold break price series, with three days of in-sample forecasts and ten days of out-of-sample forecasts. The real and estimated values were closely aligned, demonstrating the efficiency of **ARIMAX(1,2,0)(1,1,0)** in representing the data.

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Compliance with ethical standards

Disclosure of conflict of interest

The authors declare that they have no conflict of interest.